Detection of incremental and decremental bars at different locations across Mach bands and related stimuli

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Two-alternative forced-choice procedures were used to measure the detectability of bright and dark bars at various locations across luminance patterns that produced Mach bands. Detection performance was significantly affected by both dark and bright Mach bands: poor detection performance was observed at locations near, but not in, the Mach bands; relatively good detection performance at locations within the Mach bands was caused by reliable changes in the width, depth, or symmetry of the bands produced by the signal bars. The changes were apparent with signals of lower luminance than that needed for detection in the plateau regions far from the bands, but, because the cues were not sufficiently reliable to allow errorless performance, unusually shaped psychometric functions were obtained.
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1. INTRODUCTION

Mach bands are regions of brightness and darkness, not present in the stimulus but arising from the visual system's response to certain spatial distributions of luminance.[1] Mach, for example, observed ramp-like luminance distributions that changed linearly from a region of uniformly low level to one of higher level. Although the luminance distribution was that of a ramp, Mach observed that the transition from the light plateau to the top of the ramp was interrupted by a narrow bright band and that the transition from the bottom of the ramp to the dark plateau was interrupted by a broader dark band. Mach showed that these bands, Mach bands as they are now known, are not part of the objective luminance distribution but are brightness changes caused by some aspect of visual processing that has yet to be fully explained.

The chief impediment to our understanding Mach bands is the difficulty of measuring them.[1] But in spite of differences arising from different methods of measurement and among different observers, a few general characteristics have been observed when the Mach-band-generating stimuli are viewed for relatively long periods.[2–4] Increasing the maximum luminance or increasing the gradient of the ramp accentuates the differences between the bands and their neighboring plateaus as well as narrowing the width of the bright band (the width of the dark band is said to be not greatly affected by gradient changes). Decreasing the width of the ramp below a certain critical value (somewhere between 4 and 7.5 arc min of visual angle[5]) abolishes both bands; they are not usually reported at a step, although a bright band is sometimes reported.[6] Mach bands also disappear if the gradient of the ramp is made very shallow.

Several studies have attempted to quantify the appearance of Mach bands by asking observers to adjust the luminance of a comparison stimulus until it matched the perceived brightness of an adjacent region of a ramp pattern.[3,7] Although there are a number of difficulties with this technique, including considerable variability within and between individuals and the dark band's sometimes appearing darker than a comparison stimulus of zero luminance, there is general agreement, for long-duration stimuli, that the bands are asymmetrical. The bright band usually appears narrower and greater in magnitude than the dark band and is reported to be centered over the discontinuity in the gradient between the top of the ramp and the light plateau. The wider and less pronounced dark band is reported to be centered on the plateau side of the bottom edge of the ramp. Further, the measured brightness gradient between the two plateaus is always steeper than the objective luminance distribution. Mach bands are reported to appear more symmetrical if exponential luminance gradients are used instead of the standard linear ramp.[8]

The relation between increment threshold and the luminance and brightness of Mach bands has been investigated and, like the bands themselves, is reported to be asymmetrical. Detection "thresholds," obtained by adjusting a series of incremental points until they were just visible, increase in the light band, but there is no minimum in the threshold at locations within the dark band.[9] Changes in the peak value of the threshold as the gradient of the ramp is changed correlate roughly with changes in the brightness of the light band. However, with the method of adjustment, the just-detectable increment seems not to be an appropriate index of brightness, since detectability appears not to be affected by the dark Mach band.[10]

One of the first and most influential models of Mach bands preceded the notion of lateral inhibition and was proposed by Mach himself.[11–15] One modern version of the model is based on the convolution of the generating stimulus with a weighting function based on the cross-sectional sensitivity of orientationally selective cortical neurons. Linear models of this sort predict, incorrectly (at least for prolonged viewing), that the bands will be symmetrical. The shortcomings of theories of this type stem from their treatment of visual processing as linear, and the asymmetry of Mach bands argues against linear processing.[16]

One suggestion to correct the problem is that an early nolinearity in the visual system modifies the input to spatial-frequency-selective channels that help determine the appearance of the stimuli. This is supported by the finding that exponential ramps are judged to have more symmetrical bands than linear ramps,[8] and odd-order power-series representations of the Mach-band stimulus can certainly be made to fit its appearance, at least as far as the appearance can be measured with brightness-matching methods. The problem posed by asymmetrical Mach bands may also be overcome if (nonlinear) divisive inhibitory lateral interactions are used instead of subtractive ones[16–19] or if luminance-dependent weighting functions are introduced.[13]

An alternative (nonlinear) approach uses constant-volume operators that dynamically alter their shape in response to the luminance level at their centers.[20–23] An array of such operators can predict something of the appearance of Mach bands without requiring any lateral inhibitory interactions. Although nonnegative weighting functions are used in this analysis, the constant-volume behavior of the weighting functions mimics some physiological measurements that indicate that the size of the antagonistic surrounds of receptive fields is reduced as the background luminance is increased, much as von Békésy's[13,14] analysis suggested. (Alternatively, of course, the physiologically observed behavior reflects something like that of a constant-volume operator and is misinterpreted as a loss of lateral inhibition with decreasing luminance.)

Mach noted that the rate of change of slope at the ramp edges in a Mach-band pattern (i.e., the second derivative of luminance with respect to distance in the direction along the generating contour) affects the width of the perceived bands. The smaller the second derivative, the smaller the size of the bands. Further, the bands appeared close to maxima or minima in the second derivative of the object. Mach proposed a model of brightness perception where the brightness at any point was related to the second derivative of the generating pattern at that point. This may appear unworkable, as at any abrupt change in generating gradient, e.g., at a ramp edge, the second derivative is undefined, but this can be overcome, as Mach noted, if the optical characteristics of the eye, which smooth any abrupt changes and ensure a finite second derivative, are taken into account. The proposal has rather a modern ring; the second (directional) derivative is readily approximated on several scales by orientationally selective cortical cells with symmetrical regions of lateral inhibition. By including cells with odd-symmetrical receptive fields to produce approximations to the first derivative of their input, we might generalize Mach's notion and try to represent the perceived pattern as a linear combination of the stimulus and the first two derivatives in the direction normal to the stimulus orientation, even though the concept of a visual system composed of only matched pairs of odd- and even-symmetrical receptive fields[24,25] is known to be incorrect.[26] The principal problem with this approach, as Marimont[16] pointed out, is that it requires the dark and light Mach bands to have the same width and to have peaks that are the same amplitude above and below the brightness of the plateaus—neither of which is the case, at least for stimuli of long duration.

Differentiation is also a critical component of a series of models called integration models.[27] These models differentiate a log transform of the objective image and then apply a threshold criterion. Gradients above the threshold are then integrated to re-form the subjective luminance distribution. This type of model can predict some brightness phenomena, e.g., the Cornsweet illusion, but not others, in particular not the appearance of Mach bands.

The most recent series of theories related to Mach bands use filters modeled on the receptive fields of cells in the visual cortex. The filters generate approximations to the first and second derivatives of the luminance profile and are used to create featural maps.[15,25,28–30] The maps then act as symbolic descriptions of salient local luminance changes signaling lines and edges, which, in turn, determine how the image is perceived. Although the models differ slightly, they do share many common features. Most of them employ filters related to odd- and even-symmetrical receptive fields and filter the image in parallel on several different spatial scales. The differences among the models lie principally in how they combine the information derived at the different spatial scales to generate a final percept. Although each model can account for a variety of brightness phenomena, there is no one model that predicts how an image will be perceived. Mostly, however, they reliably predict the conditions under which Mach bands are visible.

The following experiments explore how Mach bands behave as masking stimuli. The stimuli generating the bands are present in both observation intervals of a standard two-alternative forced-choice (2-AFC) experiment, and the signal to be detected—a narrow bar—is present in only one of the intervals. There are three main reasons for performing these experiments:

1. Previous increment-detection experiments with Mach bands as background or masking stimuli used the

equivalent of a Yes/No measurement procedure, and the results of such experiments are difficult to interpret because of the problem of differentiating between sensory effects and criterion changes,[31,32] proper two-interval 2-AFC experiments avoid this problem.

2. Determining the mechanisms underlying the appearance of even the simplest visual stimuli is difficult, and it may be that models for the detection and the discrimination of stimuli will be easier to formulate. Such models may, in turn, lead to insight into why things look as they do.

3. We will subsequently use backgrounds other than Mach bands (steps and the Cornsweet illusion), and it is useful to have data for all three backgrounds obtained by using the same measurement technique and the same observers.

2. GENERAL METHODS

Two of the authors (RWM and GBH) were required to detect horizontally oriented "signal" bars in standard 2-AFC experiments. The signal bars subtended 0.9 arc min at the viewing distance of 250 cm and extended across the full 4 deg of the display. The signal bars were luminance increments (or decrements) added to a background pattern. The experiments were mainly concerned with Mach bands, and the background was a stimulus for Mach bands in experiment I and either a step in luminance (experiment II) or a stimulus for the Cornsweet illusion (experiment III). In all cases the luminance in the horizontal direction was constant inside a region subtending 4 deg $\times$ 4 deg of visual angle. The 4-deg $\times$ 4-deg region appeared in an otherwise dark area subtending 6.8 deg $\times$ 5.5 deg. Within the 4-deg $\times$ 4-deg region, there were only vertical changes in luminance (in order that luminance change at the line rate of the display and not at its pixel rate). Thus, for the Mach-band stimulus, the ramp modulation of the masking pattern was identical along any vertical slice through the stimulus. Luminance was measured with a Gamma Scientific photometric telescope calibrated against a beta radiation source. The mean luminance of the display was 78 cd/m^2.

The display was a Mitsubishi FR8905SKHKL color monitor driven at a frame rate of 152 Hz with no interleaving. The display was linearized, and the stimuli, first generated as a 256-point array of luminance values (one for each line of the display), were produced by connecting two of the four independent 8-bit digital-to-analog convertors of a NuVista frame store through a passive attenuator to the "green" gun of the display.[33] With the digital-to-analog convertors added in a ratio of 7 to 1, we achieved approximately 12-bit precision in the linearized image by independently selecting the two 8-bit bytes required for the luminance of each line. Except for the two 400-ms observation intervals on each trial, the 4-deg $\times$ 4-deg stimulus region remained a uniform field of 78 cd/m^2.

Before trials began, the observers viewed the uniform field in a dim background for a few minutes' adaptation. There were two observation intervals on each trial separated by an 800-ms pause. The background stimulus, gated on and off rectangularly in time, was presented during both 400-ms observation intervals. The subjective appearance of Mach bands varies greatly with the duration of the stimulus generating them. They require little time to develop, however, and are reported at stimulus durations of 10 ms.[10,34] We chose the brief 400-ms duration to reduce the appearance of afterimages.

On each trial the observers had to choose which of the two presentations of the background also contained the signal. On each trial the signal was as likely to be in the first as in the second observation interval. The spatial location at which the signal might appear was indicated during both observation intervals by inward-pointing arrows on both sides of the display. The horizontal shafts of the arrows, generated by using the "red" gun of the display, were clearly visible and lasted for the duration of each observation interval. The arrows were also present during the 600-ms warning interval that preceded the first observation interval by 200 ms. During each observation interval, the red gun was also used to generate faint numerals located on each side of the stimuli outside the arrows and indicating observation interval 1 or 2. After a 1000-ms answer interval, during which both observers pressed buttons to indicate the interval thought to have contained the signal, a larger version of one of the numerals appeared at the bottom of the display to indicate whether interval 1 or interval 2 had contained the signal.

Fifty-five trials with signals of the same size and at the same location were run in a continuous block, with the first five trials serving only for practice. The size of the signal was then changed in order to generate five- or six-point psychometric functions relating the percentage of correct responses to the size of the signal. Then the location of the bar was changed and the process repeated, beginning in the lower-luminance region at the top of the display and working downward. Finally, the whole experiment was repeated, working upward from the higher-luminance region at the bottom of the display, so that ultimately there were 100 observations for each signal size at each bar location for each observer.

3. EXPERIMENT I: MACH BANDS

A. Method

The vertically oriented cross-sectional profile of the background stimulus used in this experiment consisted of horizontally oriented "dark plateau" (at the top of the display) with a luminance of 40.25 cd/m^2 and subtending 4 deg $\times$ 1.32 deg of visual angle at the observers' eyes. At the bottom of the display, there was another horizontally oriented "bright plateau" of 114.75 cd/m^2, also subtending 4 deg $\times$ 1.32 deg. A linear luminance gradient (subtending 4 deg $\times$ 1.32 deg) linked the two plateaus. In experiment I the observers were required first to detect a bar produced by a luminance increment at various locations in the background stimulus and then to detect decremental bars.

B. Results

Psychometric functions were measured at 20 locations on the background stimulus. Most of the measurements were made at locations near or between the bright and dark Mach bands; none was made near the top or the bottom of the display. Figure 1 shows several psychometric functions in which the percentage of correct responses is plotted as a function of the luminance increment (shown on a logarithmic axis); results for RWM are given in Fig. 1(a), and those for GBH are given in Fig. 1(b). The vertical location of the incremental bar to be detected is shown as a parameter in degrees of visual angle from the top of the display.

The error bars on the left-hand side of Figs. 1(a) and 1(b) indicate ±1 standard deviation from 90%, 75%, and 60% correct responses. The error bars are based on the assumption that the number of correct responses in 100 trials is binomially distributed. (The ends of the error bars can also be reflected through the psychometric functions to provide estimates of the variability associated with a threshold increment value.)

Results from regions near the Mach bands need to be considered separately from those from regions remote from the bands. The latter are simple in that psychometric functions from locations remote from the Mach bands (which lie near 1.3 and 2.7 deg) are roughly parallel on semilogarithmic coordinates. For positions remote from the Mach bands, performance is readily captured by defining a threshold performance—75% correct, say—and interpolating in the psychometric functions to determine the size of the corresponding threshold increment.

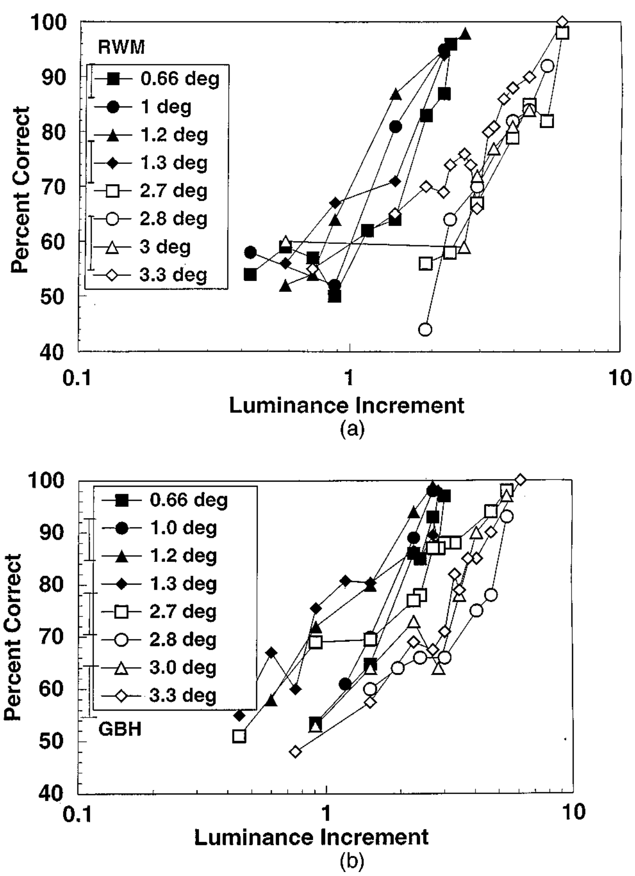


Fig. 1. Psychometric functions—the percentage of correct responses as a function of the size of the signal increment (cd/m^2 on a logarithmic scale). The results are for the detection of increments at the different locations along the Mach-band pattern for a single observer [RWM in (a) and GBH in (b)]. Location, measured from the topmost edge of the horizontally oriented dark plateau, is indicated (in degrees) by the symbol shown in the inset. The filled symbols show data from locations in the darker regions; open symbols those for locations in the brighter regions. The error bars at the left-hand edge of the figure give the range ±1 standard deviation around 60%, 75%, and 90% correct on the assumption that the number correct in 100 observations was binomially distributed.

Because the psychometric functions from locations remote from the Mach bands are parallel on semilogarithmic coordinates, functions showing the increment threshold on a logarithmic axis as a function of location would have the same shape for any reasonable choice of threshold performance level. For regions remote from the Mach bands, there is, then, nothing surprising in the results: Increments are harder to see when they occur on the bright plateau than when they occur in the dark plateau, and the ratio of the average 75% correct threshold increment to background luminance in the dark plateau (0.035 for RWM, 0.036 for GBH) is only slightly larger than the ratio in the bright plateau (0.028 for both observers).

The results from regions in or near the Mach bands are not so simple. Figure 2 allows comparison of psychometric functions for detecting incremental bars at 0.66 deg (the center of the dark plateau) and in the center of the dark Mach band (located at 1.3 deg at the low-luminance edge of the ramp). Figure 2(a) shows the results for RWM, and Fig. 2(b) shows the results for GBH; both figures show the percentage of correct responses as a function of the size of the increment (cd/m^2)—both axes are linear. A striking feature of both sets of results is that the shape of the functions in the band differs markedly from that of the functions on the plateau. At performance levels above 71% correct for RWM (81% for GBH), the functions from the two regions are almost identical. Performance for RWM may even be slightly better on the dark plateau than in the dark Mach band. Below some critical performance level, however, both observers are nearly a factor of 2 better at detecting the incremental bar when it is centered in the dark Mach band than when it lies on the dark plateau.

Consequently, the pattern of results that appears when the threshold increment is plotted against location will depend on the performance level chosen to be the "threshold." And the differences are not trivial: At low performance levels, the increment threshold is lower in the dark Mach band than on the dark plateau; at intermediate levels there is no difference in the thresholds; at high performance levels, thresholds may be slightly higher in the dark Mach band than on the dark plateau. The psychometric functions are not parallel on the linear coordinates of Fig. 2, nor are they parallel on semilogarithmic coordinates. Indeed, if, as it appears from Fig. 2, the psychometric functions cross (and, in the technical sense of Levine,[35] just being equal at some point and different at another is crossing), there is no nontrivial transformation of the luminance increment axis that can make them parallel. The same characteristics were observed in psychometric functions obtained near the bright Mach band.

Thus the different shape of the psychometric functions at different points along the background means that performance cannot be described properly by a single thresh-

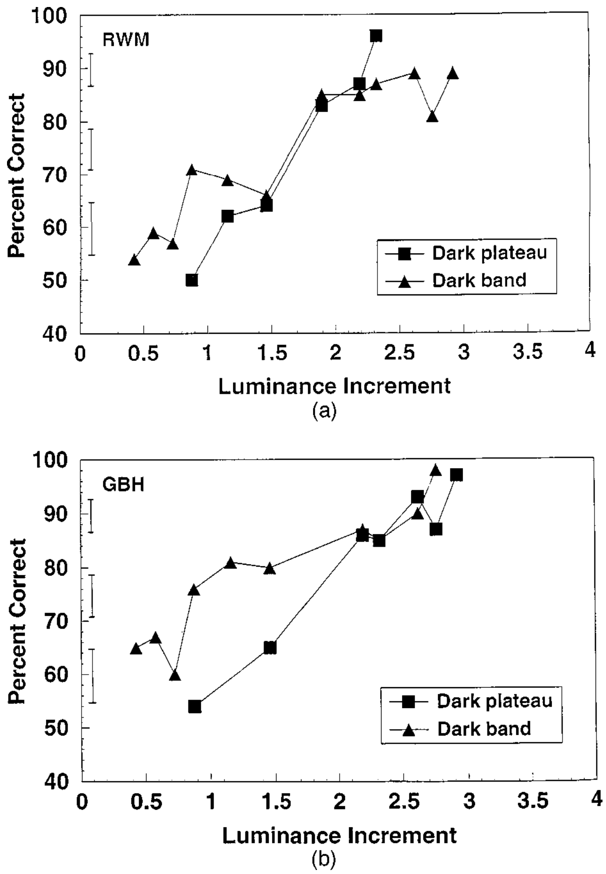


Fig. 2. Percentage of correct responses as a function of the signal increment (cd/m^2 on a linear scale) for a location in the center of the dark plateau (0.66 deg) and for a location near the center of the dark Mach band (1.33 deg) [for RWM (a) and GBH (b)]. Each data point is based on 100 observations, and the error bars at the left-hand edge of the figures give the range ± 1 standard deviation around 60%, 75%, and 90% correct on the assumption that the number correct in 100 observations was binomially distributed.

old performance level and that information about the entire psychometric function is required. To this end we provide contours corresponding to increments producing performances of 60%, 75%, and 90% correct. The results are shown separately for each observer in Figs. 3 and 4.

Figures 3(a), 3(b), and 3(c) (RWM) show the logarithm of the luminance increments that correspond to 60%, 75%, and 90% correct responses, respectively, as a function of location. [Figures 4(a), 4(b), and 4(c) show the corresponding results for GBH.] The closed rectangle under each abscissa near 1.3 deg shows the location of the dark Mach band, and the open rectangle near 2.6 deg shows the position of the bright Mach band. (The locations were determined by setting the signal increment to zero, repeatedly presenting the Mach-band stimuli for the 400-ms duration used in experiment I, and requiring each observer to adjust the faint red arrows, which in experiment I indicated the location of the signal bar, to indicate the upper and lower edges of each band. The locations shown are the average of at least six settings for each edge.)

Figures 3(a) (RWM) and 4(a) (GBH) show the logarithm of the luminance increment corresponding to 60% correct as a function of location along the background stimulus. Location is measured in degrees of visual angle from the top of the display. The luminance ramp begins at 1.33 deg and ends at 2.67 deg. For both observers, performance improves as the incremental bar moves along the dark plateau toward the dark Mach band; the luminance increment corresponding to 60% correct is considerably lower in the dark bar than on the dark plateau. The performance of both observers is poor just below the edge of

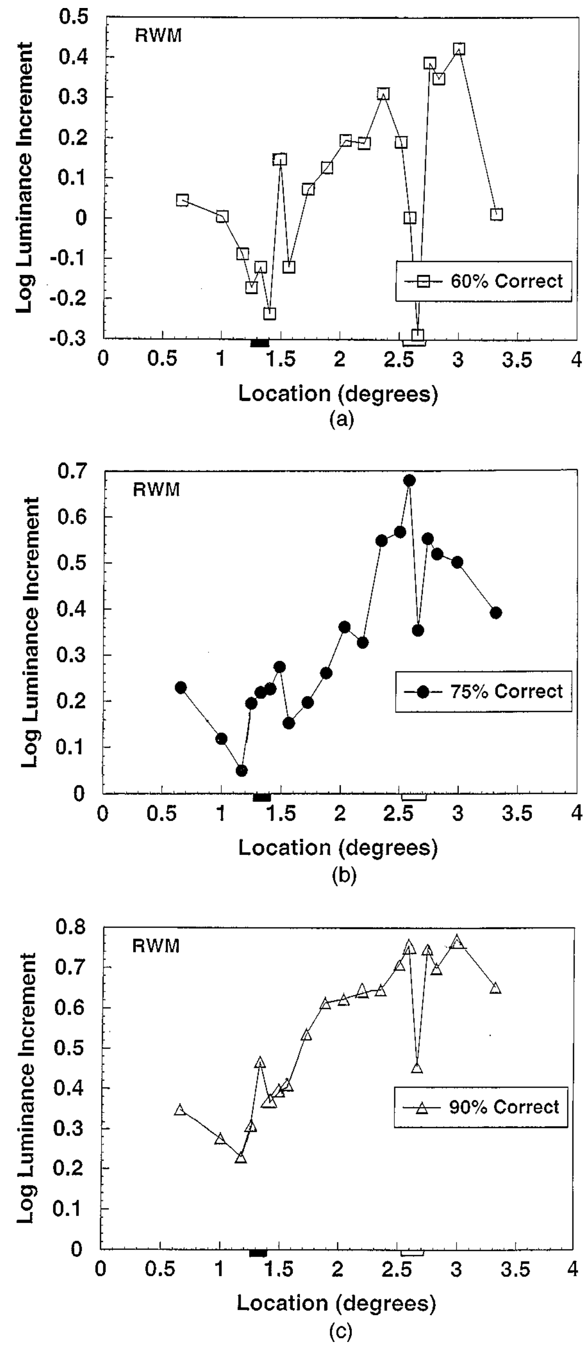


Fig. 3. Logarithm of the increment levels corresponding to (a) 60%, (b) 75%, and (c) 90% correct for locations along the Mach-band pattern. The increment values are taken from linear interpolation in the psychometric functions for observer RWM. The closed rectangle below the x axis near 1.33 deg represents the location of the dark Mach band, and the open rectangle near 2.67 deg represents the location of the bright Mach band.

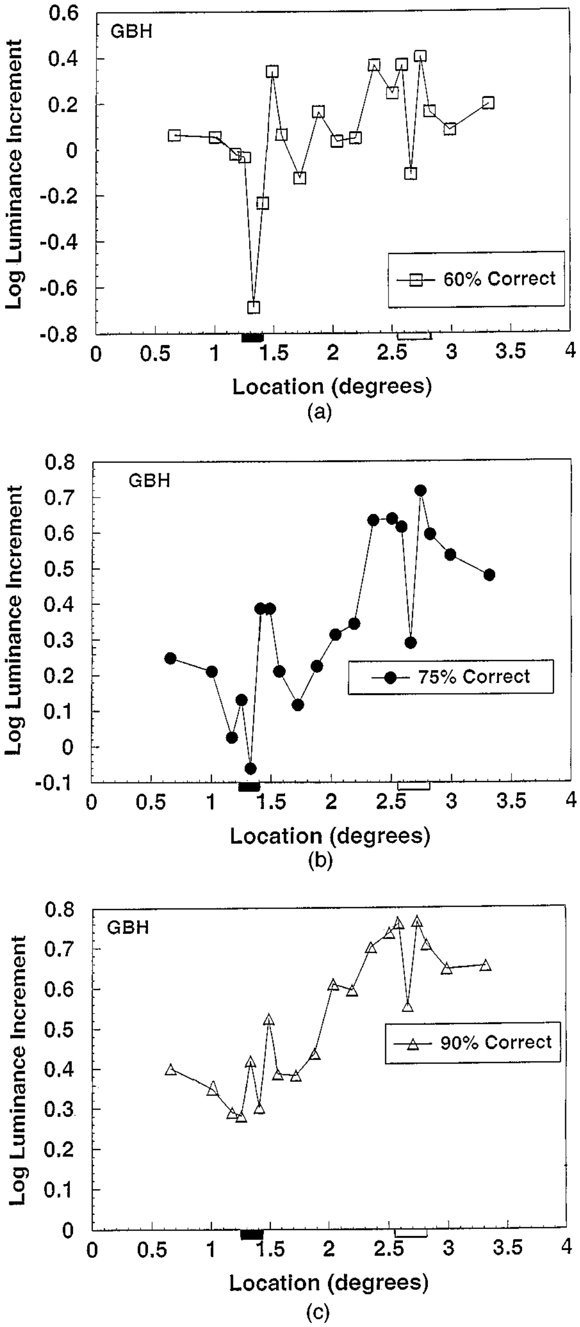


Fig. 4. Same as Fig. 3 but for GBH.

the dark Mach band, then rises gradually along the luminance ramp. Both observers show a spike of good performance for bars in the bright Mach band, and for both observers, performance near the edges of the bright Mach band is worse than that in the center of the bright plateau.

A similar pattern in the results at 75% correct appears in Figs. 3(b) (RWM) and 4(b) (GBH), except that RWM's best performance occurs just above the dark Mach band and is only slightly better than his performance on the dark plateau.

For the 90% contour, performance in the dark Mach band is only slightly better than that on the dark plateau. There remains the spike of good performance in the bright Mach band (also shown by one of the observers in Fiorentini *et al.*[9]), and, apart from that spike, performance near the bright band remains, for one observer at least, somewhat worse than that on the bright plateau.

Data for decrements were obtained on a coarser scale. The results for RWM are shown in Fig. 5(a), where the luminance change (the absolute value of the decrement) is shown on a linear scale against location. Figure 5(b) shows similar results for GBH, but the logarithm of the absolute value of the decrement is plotted. For both observers there appears to be a spike of good performance in the dark Mach band; GBH produces his best decrement detection there. The spike for RWM, on the other hand, leaves his performance in the dark Mach band no better than his performance on the dark plateau. Both observers show a spike of good detection performance for decrements in the bright Mach band in only the 60% contour; at higher performance levels, they appear to perform worst when detecting decrements in the bright Mach band.

In summary, detection is affected by the bands in the same way for both observers. The bands appear both to raise and, near the center of the bands, to *lower* thresholds relative to performance in the surrounding areas of

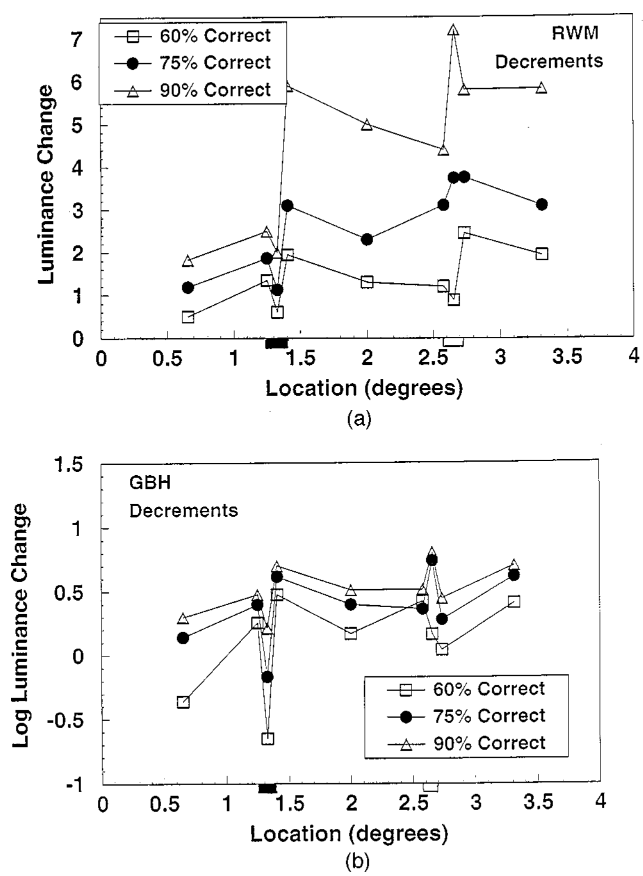


Fig. 5. Absolute value of luminance decrements corresponding to 60%, 75%, and 90% correct as a function of location along the Mach-band pattern. The size of the decrement [cd/m^2 on a linear scale in (a) for RWM; the logarithm of the size of the decrement in (b) for GBH] are linearly interpolated from the psychometric functions. The closed rectangle below the x axis near 1.33 deg represents the location of the dark Mach band, and the open rectangle near 2.67 deg represents the location of the bright Mach band.

the ramp and the plateaus. Because of the unusual shape of the psychometric functions in and near the bands, improvement in performance is greater when performance thresholds corresponding to lower performance levels are considered.

C. Discussion

To see why the detectability of increments and decrements changes so markedly in the region of the Mach bands, consider the reports of the observers. On the plateaus or in the center of the ramp, the signal, when visible, appears as a bright (increments) or dark (decrements) bar across the stimulus. But in the region of the bands, this is not the case. In the region of either band, at low signal levels, the observers reported that they could see neither brightness increments nor decrements at the location of the signal. Instead, they detected the signal by looking for reliable changes in the width, the depth, or the symmetry of the Mach bands. Adding an increment in the center of the dark Mach band, for example, broadened the appearance of the band; adding an increment toward either edge of the dark Mach band made it noticeably asymmetrical. Ratliff *et al.* report similar effects.[18]

The cues, which we observed only near Mach bands, were not easy to see, and our using them rarely led to errorless performance; rather, such cues were useful up to some performance level, after which further increases in the magnitude of the signal appeared to have little effect on the appearance of the bands (and, consequently, little additional effect on performance) until the luminance reached a level where the signal bars could be seen as such (at least seen some of the time). At this point, increasing the magnitude of the signal produced further increases in performance. The changes in cue are reflected in the shape of the psychometric functions obtained near the Mach bands.

4. GENERAL DISCUSSION

The problem posed by our results is to explain the variations in the detectability of increments and decrements at different locations in the background stimulus. In the regions in and around the Mach bands, performance changes in ways very different from that predicted by Weber's law based either on luminance or on brightness, and we ought to be able to explain those changes.

A. Comparisons with Other Studies

Increment detection improves markedly in the central region of the bright Mach band for both observers. This is consistent with the behavior of one of the observers in Fiorentini *et al.*,[9] who asked subjects to adjust, until no longer visible, the luminance of a series of incremental points aligned with the generating stimulus. Apart from the spike of good performance for one observer, Fiorentini *et al.* reported that the "threshold" luminance followed the subjective brightness distribution for the bright part of the field (i.e., the bright band and plateau) but that increment detection failed to show any effect of the dark band. They observed no spike of good performance in the dark Mach band. With 2-AFC procedures, however, increments in the dark Mach band are shown to be detectable at even lower levels than those detectable on the uniform field of the same luminance. A similar phenomenon appears to occur in the bright band. The result is due to a change in the cue used by the observers in detecting signals near Mach bands, so that what is measured arises not from the visibility of the signal but rather as detection of predictable changes in the subjective appearance of the Mach bands—changes that are correlated with nearby (but, as such, invisible) luminance increments or decrements. The disproportionate worsening in performance seen in regions adjacent to the trough or the peak of the bands corresponds to areas where both the signal bars and the deformations of the bands are hard to see and thus detection performance suffers.

The shape cues are likely to remain undetected in single-interval Yes/No experiments, as there is no easy way for the observers to compare the shapes of the Mach bands with and without a signal. The pattern of results with the use of a Yes/No procedure is very similar to that which we observe along a high threshold cut—at 90% correct, say. Thus the observers of Fiorentini *et al.*[9] using the method of adjustment appear to have adopted a conservative decision strategy with a strict criterion, the consequence of which was to miss some of the features of detection, particularly near the dark Mach band.

The question now becomes: Why are Mach bands so sensitive to the addition of increments at certain locations in the luminance profile? Ratliff *et al.*[18] noted that the addition of a barlike perturbation to a Mach-band pattern narrowed the width of Mach bands in a manner dependent on the contrast of the bar and its distance from the band. The closer the bar to the band, and the greater its contrast, the greater the effect. However, the effect was independent of the width or the polarity of the bar itself. Our observations differ; an increment in the center of a band widens a dark Mach band and narrows a bright one. Decrements do exactly the opposite. Both increments and decrements can affect the apparent symmetry of both bands, and it is the asymmetrical appearance that our observers use in detecting bars in, but not centered on, either Mach band.

B. New Models

A novel approach to understanding spatial vision has been developed and explored by Cornsweet and Yellott.[20–23] They consider the effects of constant-volume operators, which are treated as (nonlinear) point-spread functions in a convolution-like equation that integrates the contributions from different points in the stimulus "object" to give a point in the "image." Two ways of viewing normal convolution consider either (1) the contribution of each point in the object to each point in the image [through the (single) point-spread function] or, equivalently, (2) the contribution that each point in the image accepts from each point in the object [through a (single) weighting function or receptive-field sensitivity, suitably folded]. The nonlinear constant-volume operators are amenable to the elegant analysis of Cornsweet and Yellott in only the first framework; the receptive fields corresponding to Cornsweet and Yellott's constant-volume point-spread operators would vary from location to location, being distorted in both shape and sensitivity by the luminance pattern to which they are exposed.

Obversely, in a more conventional approach, one might envisage mechanisms whose gain and receptive-field area are both related to the luminance at the center of the receptive field. By analogy with Cornsweet and Yellott, one might consider purely excitatory, circularly symmetrical receptive fields. Both the "gain" of a receptive field (the height at its center in a three-dimensional plot of its sensitivity as a function of location) and the area of the receptive field might depend on the luminance at the receptive fields's center (in the same way as the heights of Cornsweet and Yellott's constant-volume operators depend on the luminance at their centers). With increasing luminance the receptive-field area might shrink, and the height grow, in such a way that the volume calculated by integrating sensitivity over the receptive field is constant. (Such a mechanism is similar to that proposed by von Békésy[14] to explain why bright Mach bands often appear narrower than the dark bands.) The point-spread functions associated with constant-volume receptive fields in a convolution would, of course, be distorted in both shape and magnitude by the receptive fields to which they contribute.

Cornsweet and Yellott show their constant-volume operators to have some fascinating properties, many of which also appear in the behavior of human observers. Their analysis includes the responses both to deterministic and to stochastic stimuli.[21–23] The former are shown to be the responses to the mean values of the latter (the two responses become indistinguishable) above a very moderate luminance level well below that of our stimuli.

One of the characteristics of constant-volume operators is to produce Mach-band-like phenomena at a step where, of course, Mach bands are not normally seen. It therefore seemed appropriate to measure masking near a step of the same contrast as that of our Mach-band-generating stimulus using the same equipment and the same observers.

5. EXPERIMENT II

A. Method

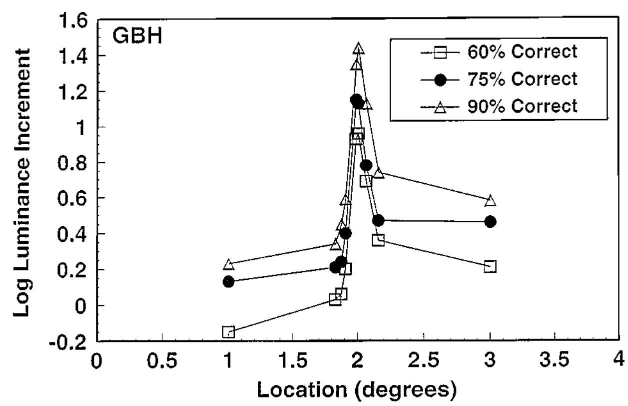
The equipment, the stimulus generation, and the psychophysical methods were those described in Section 2. The background against which incremental or decremental bars were detected, however, was a step in the center of the display where the luminance changed from the horizontally oriented dark plateau (40.25 cd/m^2), which now subtended 4 deg × 2 deg at the top of the display, to the horizontally oriented bright plateau (114.75 cd/m^2), which also subtended 4 deg × 2 deg but at the bottom of the display.

B. Results

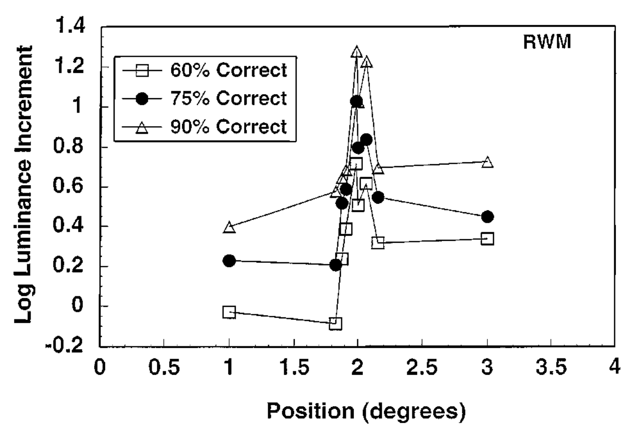
Figure 6(a) shows for GBH the logarithm of the increments corresponding to 60%, 75%, and 90% correct as a function of location. At all three levels, performance is worse on the bright side of the step than on either plateau. This observer was unable to achieve 90% correct detection of the increment at 1.4 arc min to the dark side of the step with the largest increment that we could produce. Figure 6(b) shows the results for RWM.

In spite of differences in measurement technique and in probe stimuli, the results are similar to those of previous experiments with step masking stimuli.[36,37] The pattern of results is different from those with the ramps of experiment I in several respects: Only one spike of good performance appears (for RWM but not GBH), and that spike occurs right at the step.

The results for increment detection and decrement detection (not shown) are almost exactly the same for RWM, including the spike of better performance on the step. His results are similar to those of Limb and Tulunay-Keesey[37] (obtained with brief signals and simultaneously occurring backgrounds) in that there is a small region of good performance near the step. Their observers' good performance lies approximately 1 arc min from the step, but there are important differences between our study and theirs in mean luminance and in the way luminance was changed. RWM reports seeing a very narrow bright Mach band in response to the step stimulus, and, at the spike of better performance, he reports using the width of the bright band as a cue. The only difference between his performance with decrements and his performance with increments is that he is approximately 10%



(a)



(b)

Fig. 6. Logarithm of the increments corresponding to 60%, 75%, and 90% correct for locations along a step pattern. The step (from 40.25 to 114.75 cd/m^2) occurred in the center of the display at 2 deg. The increment values were linearly interpolated from the psychometric functions (100 observations per point) for observer GBH. (b) Same as (a) but for RWM.

better at decrement detection. The decrement detection of GBH is similarly better than his increment detection and parallels his behavior with increments. The superior performance with decrements has been previously observed for a briefly flashed signal on static backgrounds.[37]

C. Discussion

As with measurements of appearance[7] and in the detection data,[10,38] there is little evidence for dark Mach bands in detection performance near a step. Detection performance is bad near the step on its higher side, but, unlike the results with the 1.33-deg ramp of the same contrast, and the results with brief signals and continuously presented steps,[37] there is only a slight worsening of performance on the lower side.

The response of constant-volume operators to a step crossing a sufficiently large mean luminance is identical in both plateau regions remote from the step because the constant-volume operators ensure that the output in response to any nonzero uniform field is independent of its mean luminance.[20,21] However, the response to a narrow stimulus, or spot, in these regions, does depend on the background luminance and exhibits Weber's law, which also approximates our observers' behavior in the plateau regions far from the step transition (but not that of Fiorentini and Zoli's observers at a step[36]).

The Mach bands produced by constant-volume operators at a step are odd-symmetrical about the step in that they extend as far below the response to the dark plateau on the low-luminance side of the step as they extend above the response to the bright plateau on the high-luminance side. The locations of the maximum and the minimum in the output are symmetrical about the step, and, for a mean luminance as great as ours, the magnitude of the Mach bands depends only on the ratio of the luminances across the step. With Yellott's likely scale factor ($\sigma = 100$) and our ratio of luminances, the extrema for the "Mach bands" in the output of the constant-volume operator in response to our step lie only 10 arc min of visual angle from the step. The ratio of the difference between the maximum (or minimum) response and the response far from the step is 1.65—more than half the ratio of our step. Several of our measurements are between the step and the extrema and might be expected, as with ramps, to reveal both the bright and dark bands of roughly equal magnitude predicted by the constant-volume operator, but we find no evidence of the dark band.

6. EXPERIMENT III

Our final experiment measured increment detection across the stimulus used to generate the Cornsweet illusion.

A. Method

The equipment, the stimulus generation, and the psychophysical methods were those described in Section 2. The background against which incremental bars were detected, however, was approximately an odd-symmetrical double exponential of the form $\mathrm{sgn}(y)\exp(-\alpha|y|) + L$, where the y origin is at the center of the display (at 2 deg in our figures) and $\mathrm{sgn}(y)$ is equal to -1 for $y < 0$ and to $+1$ for $y \geqslant 0$. The stimulus was scaled so that its contrast matched that of the ramp and step stimuli in experiments I and II, and the mean luminance of the display, L, was again 78 cd/m^2. The parameter α was chosen so that the luminance of the stimulus at locations of more than 1 deg of visual angle from the discontinuity in the center of the display differed by no more than 10% from the mean luminance. The stimulus gave rise to the Cornsweet illusion in that the region having approximately the mean luminance of the stimulus in the top half of the field appeared distinctly darker than the corresponding region in the bottom half of the field. That is, the appearance of the stimulus was approximately that of a step with something odd at the discontinuity and the edges.

B. Results and Discussion

Figure 7(a) shows for RWM the logarithm of the increments corresponding to 60%, 75%, and 90% correct as a function of location. Figure 7(b) shows similar results for GBH.

Performance certainly does not follow Weber's law with either luminance or brightness as the background, but we were surprised to see some evidence (at least in the data of GBH [Fig 7(b)]) that detection performance in the "perceptual plateaus" regions, where the luminances were very similar, is different in a way that mimics the appear-

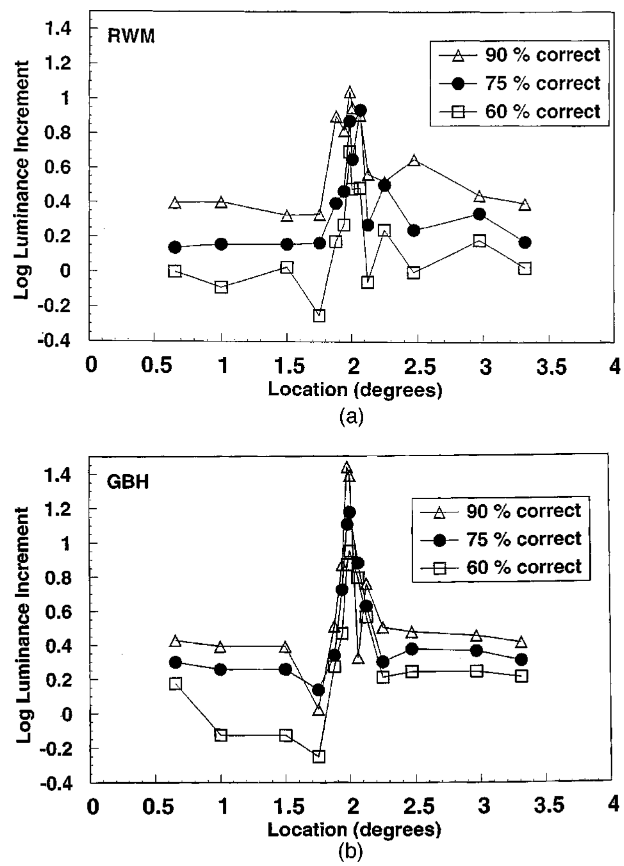


Fig. 7. Increment levels corresponding to 60%, 75%, and 90% correct for locations along a pattern producing the Cornsweet illusion. The increment values were linearly interpolated from the psychometric functions of observer RWM. (b) Same as (a) but for GBH.

ance of the stimulus. The variability of the observers' performance is high in the region of the discontinuity, where RWM again shows a spike of good performance. As with the step masker, masking is greatest on the side of the discontinuity that appears brighter.

7. GENERAL DISCUSSION (CONTINUED)

We now have two classes of phenomena to explain: the masking effect of the background stimuli and their appearance. There is, of course, no need for the appearance of the background stimuli and their masking effects to be determined by the same mechanisms. Nor need the characteristics of the three background stimuli depend on the same mechanisms—each may have characteristics evoking responses that ensure that its appearance and masking effect are determined by different mechanisms. Indeed, the diversity of models developed to characterize the appearance of Mach bands, edges, and the Cornsweet illusion may reflect an underlying diversity of mechanism. Equally, there is no need for the detection of luminance bars against these backgrounds to be governed by a common mechanism; indeed, the observers' reports from experiments in which they were detecting bars against Mach bands suggest the use of at least two different cues, if not different underlying mechanisms. Nonetheless, we should like to see how far a single mechanism might go toward predicting all our results.

The mechanism that we propose to consider begins with the Naka–Rushton[39] equation for an early luminance gain control:

$$R[I(y)] = R_{\max} I^n(y)/[I^n(y) + \alpha^n], \tag{1}$$

where R is a response indicator dependent on the intensity of the masking background, $I(y)$, which, in turn, is a function of location y; $R_{\max}$ is the maximum response, and α determines the intensity at which the response reaches half its maximum.[39] The exponent n affects a number of characteristics, particularly the slope of the response function.

Following Mach, we first modify Eq. (1) by replacing $I(y)$ in the denominator by the weighted integral $I_{\text{int}}(y)$, taken over a small range of luminances centered on y. This is the first step toward predicting detection near Mach bands. One way to make the widths of the dark and bright Mach bands differ is to make the size of the integration region depend on luminance, so that higher luminances produce smaller intervals of integration. For our 400-ms stimuli, however, the bands appear approximately equal in width, and we fixed the size of the integration window that produces the normalization term at 23 arc min of visual angle. This produces bands that subtend approximately 10 arc min of visual angle from stimuli with the characteristics of our masking pattern. Further, we used a uniform spatial weighting function for the window of integration. [The shape of the integration window might take many forms. We chose a rectangular form of constant width for simplicity; a window of the form

$$\text{window}(y) = 4|y|(W/2 - y^2)^{1/2} \sin^{-1}(2|y|/W)/\pi W \tag{2}$$

might be more appropriate for a two-dimensional analysis of our one-dimensional signals, where y is measured from the center of a circularly symmetrical window that extends for $2W$ deg of visual angle. We could also have provided a small integration region for the numerator of the equation to make the spatial characteristics match conventional receptive fields, but this could be done in many ways[40] and we preferred to keep the development simple so that the implications of each step might be apparent.]

The integral I_{int} can be scaled in various ways—we chose to divide the integral by the width of the weighting function to yield the average local luminance. In this way, the response to uniform fields of different luminance would be exactly as the Naka–Rushton equation [Eq. (1)] predicts because, with uniform fields, the average luminance is equal to the luminance. Further, because of the limitations of the Naka–Rushton equation in predicting human behavior, we followed Kortum and Geisler[41] in introducing both multiplicative (m) and subtractive (s) components into Eq. (1):

$$R[I(y)] = R_{\max}\{m[I(y) - s]\}^n/(\{m[I_{\text{int}}(y) - s]\}^n + \alpha^n), \tag{3}$$

where I is now retinal illuminance in trolands. Both[41] the multiplicative (m) and subtractive (s) components depend on the local average illuminance $I_{\text{int}}(y)$: s is proportional to the average illuminance, i.e.,

$$s = gI_{\text{int}}(y), \tag{4}$$

where g is 0.891; and, for large values of $I_{\text{int}}(y)$, m is inversely proportional to the average illuminance, i.e.,

$$m = b_0/[I_{\text{int}}(y) + b_0], \tag{5}$$

where b_0 is 59.05.

In response to uniform fields, Eq. (3) becomes Kortum and Geisler's Eq. (2) and thus captures a number of luminance and adaptation effects that depend on mean luminance.[41] Following Kortum and Geisler, we use an expansive exponent n, equal to 2.0, a "half-saturation" constant α, equal to 100, and $R_{\max}$ equal to 300.

In nonuniform fields the local average illuminance differs from illuminance, and the modified Naka–Rushton equation that uses a local average in the normalization [i.e., in the denominator of Eq. (3)] produces some interesting predictions including Mach bands.

Equation (3) may seem unduly complicated. Indeed, because our stimuli have relatively low contrast, we could ignore at least two of the luminance-dependent terms. But, without the complications, in wider contexts even the effects of mean luminance on detection thresholds cannot be predicted, and it seemed sensible to include as much predictive power as possible.

Equation (3) predicts both Mach bands in response to ramp stimuli and an asymmetrical response to steps—a much bigger response is predicted close to but on the bright side of the step. However, when the response to the stimulus is determined by a nonlinear equation such as Eq. (3), it is not possible to predict the masking effect of a given stimulus merely by looking at the response to the background or the masking stimulus alone. [The constant-volume operators of Cornsweet and Yellott,[20–23] for example, predict the same (unit) response to any nonzero uniform field, but, in terms of the response to an increment superimposed on a uniform field, different backgrounds have very different effects. With constant-volume operators a local increment must be proportional to the background luminance to produce a given response, even though all the (uniform) backgrounds produce the same (unit) response.]

The nonlinear characteristics of Eq. (3) mean that it cannot be assumed that a small probe, such as our narrow incremental and decremental bars, produces only small perturbations that neither depend on nor interact with the background. (In particular, the contribution of the probe stimulus to the local average illuminance and the accelerating nonlinearity ensures that probe stimuli in the vicinity of the Mach bands affect the magnitude, the symmetry, and the width of the bands. This is not altogether a bad thing in a model, given that our observers report using such effects and given that we shall need them in the model to account for their behavior.) Consequently, to explore the predictions of this model for the masking effects of experiment I with a Mach-band-generating ramp as the background, it is necessary to determine the effects of signal bars of the appropriate 0.9-arc min width and many different luminances at a variety of locations across the background.

Some simplification of the analysis is possible because of the fixed 23-arc min width of the integration window used for Eq. (3); the addition of a probe at locations between 0 and 0.94 deg can have no effect on the predicted dark Mach band, and probes between 3.06 and 4 deg can have no effect on the predicted bright Mach band. Further, the predicted bands will not be affected by probes located on the ramp between 1.72 and 2.28 deg. At all those locations, where neither band is affected by the probe, we can safely assume that performance in detecting probes is determined by the difference between the response to the background alone and the response to the background plus probe.

A reduction in the number of calculations also results from the fact that, with the fixed width of the integration window, the response to the probe on the dark plateau will, if we neglect the edges of the display, be the same from 0 to 1.14 deg, and on the bright plateau the response to the probe cannot change from location 2.86 deg to 4 deg.

While Eq. (3) captures many of the characteristics of the appearance of the masking stimuli, it captures only some of the characteristics of the masking data. It captures the 0.39-log-unit difference in detection performance on the plateaus, for example. There is a major flaw, however, which is apparent only when the effects of the probes are studied. The defect is that a small probe in the bright Mach band produces a big response and, consequently, should be easy to see. It produces a big response because the combination of factors that produce the bright Mach band also serve to enhance the response to the probe. To correct this defect, we made the following modifications to the model.

To capture more masking behavior, we attempted to quantify a suggestion of Nachmias[42] and of Fine[43] that dark Mach bands might travel along a route that begins with "off-centered" receptive fields and that the bright bands travel along a route that begins with "on-centered" receptive fields. We assume that information travels in an off-channel centered on a point whenever the intensity at that point is less than the local average around it, i.e., whenever $I(y) < I_{\mathrm{int}}(y)$; if $I(y) > I_{\mathrm{int}}(y)$, information travels in an on-channel. The bigger the difference between $I(y)$ and $I_{\mathrm{int}}(y)$, the bigger the response.

We take the response to a uniform field to be the base input to this stage. Thus the magnitude of the difference between the response from Eq. (3) and the response that would be obtained if the luminance and the local luminance were equal is then compressed (raised to the power 0.5). The compression occurs separately in the on- and off-channels. The separate compression makes the probe hard to detect in the region of the Mach bands whenever the probe does not visibly affect the bands. This stage may, in effect, represent a contrast-gain control mechanism, and it may operate within spatial-frequency-tuned channels. The revised model now captures most of our masking effects when we compare the response at a location with the masking pattern alone and the response with the sum of the masking pattern and a probe signal of given magnitude and sign.

The separated and compressed response predicts poor detection in the region of either Mach band. To detect an increment 75% of the time should require an increment 0.3 log unit larger in the bright Mach band than on the bright plateau. The threshold should be from 0.4 to 0.7 log unit larger in the dark band than on the dark plateau, depending on the performance level chosen to define the threshold. [The prediction depends on performance level, because for increments in the dark band (and decrements in the bright band) the effect of increasing the probe magnitude at first produces a response less than the response to the background alone, but when the probe becomes sufficiently large to have a big effect on the local average illuminance, increasing the probe magnitude decreases this difference and, ultimately, changes its sign. The effect is not visible in our measurements, however, because the observers are able to detect the effect of the probe on the shape of the Mach bands with probes of smaller magnitude than the effect requires.]

The model predicts the spikes of good performance with bars in and near either band. Where the observers report using changes in the width of the bands, incremental and decremental bars produce shape changes that are on the order of 10% of the width of the bands. We cannot easily quantify the changes in the apparent symmetry of the bands, but such changes are produced in the model and our observers report using them. The model incorrectly predicts that increments should be more detectable than decrements (by approximately 0.1 log unit). It predicts approximately a 0.7-log-unit increase in thresholds at a step where our observers show approximately a 0.6-log-unit increase, and it correctly predicts that performance 12 arc min on either side of a step will be unaffected by it. It is possible that refinement of the model, once a method to eliminate the cues based on band shape has been found, will allow it to capture more of the data but not our observers' better performance with decrements.

8. SUMMARY

The following points suggest themselves:

1. The masking effects of stimuli generating Mach bands, the Cornsweet illusion, and a luminance step on the detection of narrow bars were measured in standard two-alternative forced-choice experiments.
2. The psychometric functions measured near Mach bands were unusual in shape, reflecting the different cues that were used at different signal intensities; very good detection performance in both bands reflected the observers' using changes in the appearance of the bands produced by the signals.
3. Masking near but not in either band appears to be greater than on the neighboring plateau.
4. The stimulus generating the Cornsweet illusion and the luminance step produce similar masking functions near their central discontinuities—in both cases masking is greatest on the side of higher luminance and almost independent of location near the discontinuity on the lower-luminance side.
5. A tentative model based on a modified Naka–Rushton equation and the development of Kortum and Geisler is proposed.[39,41] The modification is to use a local average in the normalizing denominator of the equation. That modification, an expansive low-order nonlinearity, and the splitting of information flow into two channels (subsequently compressed) form the basis of a model that predicts many of the masking effects produced by all three backgrounds.
6. It was not thought sensible to elaborate the model fully until either a way of quantifying the effects of the probe signals on the width, the depth, and the symmetry of the Mach bands is developed or a way of preventing the observers' using such cues is discovered.

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